

$\overline{B} \rightarrow D^{(*)}\pi^-$ beyond naive factorization in the heavy quark limit

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Abstract

Nonleptonic decays $\overline{B} \rightarrow D\pi^-$ or $\overline{B} \rightarrow D^*\pi^-$ are dominated by factorizable contributions. In the heavy quark limit, nonfactorizable contribution arises from strong radiative correction and power corrections in $1/m_b$. I calculate the decay rates for $\overline{B} \rightarrow D^{(*)}\pi^-$ at next-to-leading order in strong interaction, including nonfactorizable corrections. The result is expressed in terms of a convolution of the hard scattering amplitude and the pion wave function. The decay amplitudes in this method are independent of the gauge, the renormalization scale, and the renormalization scheme. The effects of the nonfactorizable contribution are discussed and numerical estimates are presented.

It is difficult to understand nonleptonic exclusive decays from first principles since we do not know nonperturbative effects such as the soft gluon exchange responsible for the quark confinement. As a first attempt to understand nonleptonic decays, the idea of naive factorization was introduced to evaluate the matrix elements of four-quark operators between hadronic states [1]. But it neither explains experimental data satisfactorily nor is justified theoretically. It is necessary to include somehow nonfactorizable contributions, which arise from gluon exchange between final-state mesons. There has been a theoretical improvement on the factorization approximation using the effective Hamiltonian approach [2], and the formalism was improved to next-to-leading order (NLO) in QCD by phenomenologically introducing the effective number of colors N_{eff} [3].

It is worth mentioning a technical point in applying the effective Hamiltonian to calculate decay rates. If we compute the finite part at NLO with external off-shell quarks [2], the decay amplitudes depend on the gauge choice and the renormalization scheme [4], hence unphysical. Cheng et al. [5] show that these

dependences are absent if we use on-shell external quarks. These dependences are artifacts in regulating the infrared divergence with the off-shell momenta of the external quarks, and they disappear when we choose on-shell external quarks.

Recently Beneke et al. [6] followed the idea used in exclusive processes by Brodsky and Lepage [7] to calculate the nonfactorizable contribution in the heavy quark limit. I will employ the same idea to give an improved analysis on $\overline{B} \rightarrow D^{(*)}\pi^-$ including nonfactorizable contributions.

The basic idea is that simplifications occur when the bottom quark mass m_b and the charm quark mass m_c are large compared to the strong interaction scale Λ_{QCD} with $r = m_c/m_b$ fixed. In this limit the hadronic matrix elements for $\overline{B} \rightarrow D^{(*)}\pi^-$ can be written in the form

$$\begin{aligned} \langle D^{(*)}\pi^- | J_1^\mu J_{2\mu} | \overline{B} \rangle &= \langle D^{(*)} | J_1^\mu | \overline{B} \rangle \langle \pi^- | J_{2\mu} | 0 \rangle \\ &\times \left[1 + \sum_n A_n \alpha_s^n + O\left(\frac{\Lambda_{\text{QCD}}}{m_b}, \frac{\Lambda_{\text{QCD}}}{m_c}\right) \right], \end{aligned} \quad (1)$$

where J_1, J_2 are the bilinear quark current operators in the effective weak Hamiltonian. The naive factorization corresponds to neglecting the α_s corrections and the power corrections in Λ_{QCD} . In this case the matrix element factorizes into a form factor and a decay constant. If we include higher-order radiative corrections, this naive factorization is broken. However, the corrections are calculable from first principles using perturbation theory. Furthermore the matrix elements of the current operator between \overline{B} and $D^{(*)}$ states are also calculable systematically in powers of the inverse heavy quark masses using the heavy quark effective theory (HQET) [8].

The effective weak Hamiltonian for nonleptonic decays such as $\overline{B} \rightarrow D\pi^-$ or $\overline{B} \rightarrow D^*\pi^-$ is written as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* (C_1 O_1 + C_2 O_2), \quad (2)$$

where the four-quark operators O_1 and O_2 are given by

$$\begin{aligned} O_1 &= (\overline{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha) (\overline{d}_\beta \gamma^\mu (1 - \gamma_5) u_\beta), \\ O_2 &= (\overline{c}_\beta \gamma_\mu (1 - \gamma_5) b_\alpha) (\overline{d}_\alpha \gamma^\mu (1 - \gamma_5) u_\beta). \end{aligned} \quad (3)$$

In Eq. (2), V_{cb} and V_{ud} are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, G_F is the Fermi constant, and α, β are color indices. The coefficient functions C_1 and C_2 are known at NLO.

In the pioneering work of Politzer and Wise [9], they calculated the ratio of the decay rates $\Gamma(\overline{B} \rightarrow D\pi^-)/\Gamma(\overline{B} \rightarrow D^*\pi^-)$ including the perturbative calculation of the nonfactorizable contribution. Since the coefficient functions C_1 and C_2 were known at leading order at the time, they could present only the ratio of the decay rates. But now that the Wilson coefficients are known at NLO [10], we can calculate the decay rates themselves. Here I present the NLO result for the decay rates $\Gamma(\overline{B} \rightarrow D\pi^-)$ and $\Gamma(\overline{B} \rightarrow D^*\pi^-)$.

As in other exclusive processes considered by Brodsky and Lepage [7], the matrix elements of the operators can be written as the sum of products of matrix elements between \overline{B} and $D^{(*)}$ states with the integral over the momentum fraction x of a hard scattering amplitude times the pion wave function $\phi_\pi(x, \mu)$, normalized as

$$\int_0^1 dx \phi_\pi(x, \mu) = 1. \quad (4)$$

Here x is the momentum fraction of a quark (or an antiquark) in the pion with respect to the pion momentum p_π^μ . The hard scattering amplitude can be calculated in perturbation theory. There are other contributions, for example, from the gluon exchange with a spectator quark. They may not be negligible in $\overline{B} \rightarrow \pi\pi$ decays, but they are power suppressed and negligible in this case since the spectator quark is absorbed by the charm quark.

The Feynman diagrams, which contribute to the nonfactorization, are shown in Fig. 1. Other Feynman diagrams, in which a gluon is exchanged between quarks, are attributed to either a meson wave function or a form factor. Specifically, the contribution of the gluon exchange between the quark and antiquark pair forming a pion is absorbed in the pion wave function, and the contribution of the gluon exchange between the b and the c quark is absorbed in the form factor for $\overline{B} \rightarrow D^{(*)}$.

Since we assign a certain set of Feynman diagrams to nonfactorizable contributions depending on final-state mesons, it is important to specify first which quark and antiquark form a meson. Only after the specification, we arrange the operators in such a way that they give correct results. For example, in $\overline{B} \rightarrow D\pi^-$, we use the operators in Eq. (2) as they are. This is called “charge-retention configuration”. On the other hand, in $\overline{B} \rightarrow D\pi^0$, we have to rearrange the operators by Fierz transformation first, and calculate the corresponding Feynman diagrams like Fig. 1. This is called “charge-changing configuration”. Therefore the calculation in this method is process dependent. This prescription sounds trivial in $\overline{B} \rightarrow D\pi$, but it has a drastic effect on the processes involving penguin operators.

The amplitude for $\overline{B} \rightarrow D\pi^-$ can be written in a compact form as

$$\begin{aligned} \langle D\pi^- | H_{\text{eff}} | \overline{B} \rangle &= \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* i f_\pi p_\pi^\mu \\ &\times \left(a_- \langle D | \bar{c} \gamma_\mu (1 - \gamma_5) b | \overline{B} \rangle + a_+ \langle D | \bar{c} \gamma_\mu (1 + \gamma_5) b | \overline{B} \rangle \right), \end{aligned} \quad (5)$$

where f_π is the pion decay constant. In Eq. (5), the amplitude is decomposed into two parts in which a_- remains finite, while a_+ approaches zero in the limit $r \rightarrow 0$.

The form factors for $\overline{B} \rightarrow D^{(*)}$ become simplified in the limit where m_b and m_c are large compared to Λ_{QCD} , with the ratio $r = m_c/m_b$ fixed. To leading order in m_b (and m_c), the heavy-heavy form factors can be written as

$$\begin{aligned} p_\pi^\mu \langle D | \bar{c} \gamma_\mu (1 \pm \gamma_5) b | \overline{B} \rangle &= m_b \left[(1 - r) \langle H_c(v') | \bar{h}_{v'}^{(c)} h_v^{(b)} | H_b(v) \rangle \right. \\ &\quad \left. \mp (1 + r) \langle H_c(v') | \bar{h}_{v'}^{(c)} \gamma_5 h_v^{(b)} | H_b(v) \rangle \right], \end{aligned} \quad (6)$$

where $h_{v'}^{(c)}$ and $h_v^{(b)}$ are c and b quark fields in the HQET. We can write a similar form factor for $\overline{B} \rightarrow D^*$. The corrections in powers of Λ_{QCD}/m_b , Λ_{QCD}/m_c can be systematically calculated in the HQET [8]. Also there is a detailed analysis on the form factor in $\overline{B} \rightarrow D^{(*)}$ in terms of the Isgur-Wise function at small momentum transfer $q^2 = m_\pi^2$ from the experimental results of $\overline{B} \rightarrow D\pi$ [11].

The coefficients a_\pm arise from the QCD corrections and they are given as, at NLO,

$$a_- = C_1 + \frac{C_2}{N} + \frac{\alpha_s}{4\pi} C_2 \frac{C_F}{N} F, \quad a_+ = \frac{\alpha_s}{4\pi} C_2 \frac{C_F}{N} G. \quad (7)$$

Here $C_F = (N^2 - 1)/(2N)$, and $N = 3$ is the number of colors. The quantities F and G are obtained from Fig. 1 by symmetrizing on interchange of x with $1 - x$. They are written as

$$\begin{aligned} F &= -18 + 12 \ln \frac{m_b}{\mu} - 3i\pi + 6 \ln(1 - r^2) + \int_0^1 dx \phi_\pi(x, \mu) f_1(x), \\ G &= -r \int_0^1 dx \phi_\pi(x, \mu) g_1(x). \end{aligned} \quad (8)$$

The x -dependent parts $f_1(x)$ and $g_1(x)$ have the form

$$\begin{aligned}
f_1(x) &= 3 \ln x(1-x) \\
&\quad - \frac{3}{1-x(1-r^2)} \left[\ln x(1-r^2) + r^2 \ln(1-x)(1-r^2) + i\pi r^2 \right] \\
g_1(x) &= \frac{1}{1-x(1-r^2)} \left[2 - \ln x(1-x) - 2 \ln(1-r^2) + \ln r^2 - i\pi \right] \\
&\quad + \frac{1}{\left(1-x(1-r^2)\right)^2} \left[\ln x(1-r^2) + r^2 \ln(1-x)(1-r^2) \right. \\
&\quad \left. - r^2 \ln r^2 + i\pi r^2 \right].
\end{aligned} \tag{9}$$

Eq. (9) is obtained by using the symmetry of the pion wave function $\phi_\pi(x)$ under $x \leftrightarrow 1-x$. The hard scattering amplitudes are infrared finite because the infrared divergence is cancelled when we add all the Feynman diagrams in Fig. 1. Note that $G \rightarrow 0$ as r approaches zero since g_1 behaves as $\ln r$, hence $a_+ \rightarrow 0$. And F becomes identical to the result for $\overline{B} \rightarrow \pi\pi$ for O_1 and O_2 [6].

In obtaining Eq. (7), I put all the external quarks on the mass shell and $m_u = m_d = 0$. After a straightforward calculation, I find that there is no gauge dependence with the on-shell external quarks. At NLO, the renormalization scale dependence of the hard-scattering amplitudes is cancelled by that in the Wilson coefficient functions. Furthermore, we have the same result when we use either the NDR scheme or the HV scheme. Therefore we have the decay amplitudes independent of the choice of the gauge, the renormalization scale, and the renormalization scheme. The detailed proof will appear in a forthcoming paper.

It is interesting to note that imaginary parts appear through final-state interactions. Specifically the imaginary part comes from the gluon exchange of a quark and antiquark pair in π with the c quark. In $\overline{B} \rightarrow D^{(*)}\pi^-$, the relevant CKM matrix elements V_{ud} and V_{cb} are almost real. But in other B decays in which CP violation is studied, this strong phase should be disentangled before we try to get any information on the CKM matrix elements. Here I stress that the strong phase is calculable in perturbation theory. However, there is a caveat in numerical analysis on the imaginary part since it is sensitive to the renormalization scale at NLO. The real part is independent of the renormalization scale since the renormalization scale dependence of the hard scattering amplitude is cancelled by the Wilson coefficients at NLO. However, there is no such cancellation for the imaginary part since the Wilson coefficients at NLO are real. Therefore the imaginary part is sensitive to the renormalization scale at this order.

If we use the leading-order pion wave function $\phi_\pi(x, m_b) = 6x(1-x)$, we get

$$a_- = 1.062 + 0.030i, \quad a_+ = 0.00056 - 0.0041i, \tag{10}$$

Table 1

The QCD coefficients $a_{\mp}(D\pi^-)$ at NLO with $r = 0.3$ for three different renormalization scales μ with $m_b = 4.8$ GeV. The values in the brackets are those with $r = 0$ for comparison. And the values in the parentheses are the leading order values.

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2m_b$
$a_-(D\pi^-)$	$1.082 + 0.056i$	$1.062 + 0.030i$	$1.043 + 0.016i$
	$[1.084 + 0.047i]$	$[1.064 + 0.025i]$	$[1.044 + 0.014i]$
	(1.062)	(1.031)	(1.014)
$a_+(D\pi^-)$	$0.0010 - 0.0077i$	$0.00056 - 0.0041i$	$0.00030 - 0.0022i$
	(0)	(0)	(0)

with $\mu = m_b = 4.8$ GeV, and $r = 0.3$. In the limit $r \rightarrow 0$, $a_- = 1.064 + 0.025i$ while $a_+ = 0$. The coefficients $a_{\mp}(D\pi)$ are listed for different values of the renormalization scales in Table 1. The real part is insensitive to the change of the renormalization scale, while the imaginary part is not. When r is varied from 0.2 to 0.4 with $\mu = m_b$ fixed, a_- is in the range $(1.063 + 0.027i, 1.062 + 0.033i)$ and a_+ is in the range $(-0.00063 - 0.0035i, 0.0018 - 0.0041i)$. From this observation, we can conclude that numerically a_+ is negligible compared to a_- , and a_- is insensitive to r for $0.2 \leq r \leq 0.4$. a_- increases about 3% compared to the leading-order value, verifying that the factorizable contribution dominates in $\overline{B} \rightarrow D^{(*)}\pi^-$.

In predicting decay rates, the main uncertainty comes from the form factor for $\overline{B} \rightarrow D^{(*)}$. There are several ways to parameterize the Isgur-Wise functions and I employ the NRSX model [12] and the results from Ref. [11] to evaluate the decay rates. The branching ratios are given as

$$\begin{aligned} \text{Br}(\overline{B} \rightarrow D\pi^-) &= (3.1 \pm 0.7) \times 10^{-3}, \\ \text{Br}(\overline{B} \rightarrow D^*\pi^-) &= (2.9 \pm 0.5) \times 10^{-3}, \end{aligned} \quad (11)$$

which are consistent with the experimental results $(3.1 \pm 0.4 \pm 0.2) \times 10^{-3}$ and $(2.8 \pm 0.4 \pm 0.1) \times 10^{-3}$ respectively.

The decay rates $\Gamma(\overline{B} \rightarrow D^{(*)}\pi^-)$ are parameterized in terms of a single parameter a_1^{eff} in Ref. [13], which is fit to experimental data. In this approach, this is not a parameter, but a process-dependent quantity calculable in perturbation theory. $|a_1^{\text{eff}}|^2$ corresponds to $|a_- + a_+|^2 = 1.131$ for $\overline{B} \rightarrow D\pi^-$, and to $|a_- - a_+|^2 = 1.128$ for $\overline{B} \rightarrow D^*\pi^-$. And we do not have to introduce an effective number of colors N_{eff} [3] to parameterize the nonfactorizable contributions. We fix the number of colors at $N = 3$.

Here I present the nonleptonic decay rates for $\overline{B} \rightarrow D^{(*)}\pi^-$ in the heavy quark limit in which the nonfactorizable contributions are calculated in perturbation

theory at NLO. This calculation is an improved one compared to previous results since we include the momentum-dependent part in the decay amplitudes, which was not included so far, as well as the momentum-independent part. However, the result still depends on models. The decay rates depend on the pion wave functions which reflect our ignorance of purely nonperturbative effects. If we consider pions only, there are other independent studies on the shape of the pion wave functions using the QCD sum rules, and the light-cone formalism, etc. [14,15]. Therefore we can be reasonably sure of the form of the pion wave function. But if the light mesons in the final state include K , K^* , η , η' , or ρ in this framework, the model dependence on the meson wave functions can be severe. In this case, the experimental data should be used as an input to determine the wave functions and predict other decay modes.

The main point of this paper is to show that nonfactorizable contributions are calculable from first principles using perturbation theory in the heavy quark limit. However, in color-allowed decays such as $\overline{B} \rightarrow D^{(*)}\pi^-$, the nonfactorizable contribution is small. In color-suppressed decays such as $\overline{B} \rightarrow D\pi^0$, it will be significant. A complete analysis for $\overline{B} \rightarrow D\pi$ will be presented in a future publication. It will be also interesting to apply this technique to various B decays such as the decays into two light mesons. An extensive study on the B decays into two light mesons is in progress.

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Figure caption

Figure 1: Feynman diagrams for nonfactorizable contribution. The dots denote the operator O_2 .

